Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

Shortest spanning trees. Prim's algorithm.

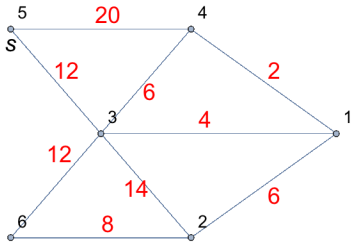
1. When will S = E at the end of Prim's algorithm?

3. What is the result of applying Prim's algorithm to a graph that is not connected?

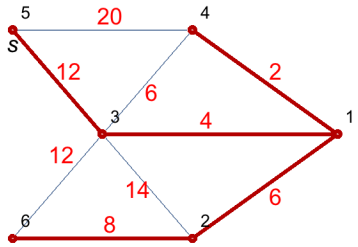
6 - 13 Find a shortest spanning tree by Prim's algorithm.

7. Problem represented by a diagram.

```
g1 = Graph[{5 ↔ 4, 4 ↔ 1, 1 ↔ 2, 2 ↔ 6, 6 ↔ 3, 3 ↔ 5, 3 ↔ 1, 3 ↔ 2, 3 ↔ 4},
  VertexLabels → "Name", VertexCoordinates ->
   {{0, 0}, {2.6, 0}, {4.7, -1.5}, {2.6, -3}, {0, -3}, {1.3, -1.5}},
  EdgeWeight → {20, 2, 6, 8, 12, 12, 4, 14, 6},
  Epilog → {{Text[Style["s", Medium], {0, -0.2}]},
    {Red, Text[Style["6", Medium], {3.8, -2.4}]},
    {Red, Text[Style["14", Medium], {1.8, -2.3}]},
    {Red, Text[Style["12", Medium], {0.7, -1.8}]},
    {Red, Text[Style["4", Medium], {2.8, -1.3}]},
    {Red, Text[Style["8", Medium], {1.4, -2.8}]},
    {Red, Text[Style["12", Medium], {0.8, -0.6}]},
    {Red, Text[Style["20", Medium], {1.4, 0.2}]},
    {Red, Text[Style["2", Medium], {3.8, -0.6}]},
    {Red, Text[Style["6", Medium], {2, -0.9}]}},
  ImageSize → 200, ImagePadding → 10]
```



```
FindSpanningTree[g1];
HighlightGraph[g1, %, GraphHighlightStyle → "Thick"]
```



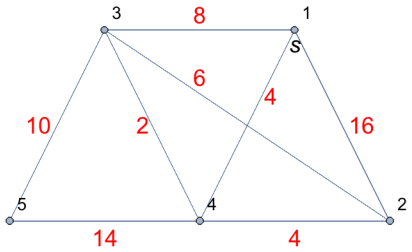

```
size[g_] := With[{edges = EdgeList[FindSpanningTree[{g, 1}]]},
  Total[PropertyValue[{g, #}, EdgeWeight] & /@ edges]]
N[
 size[
  g1]]
```

```
32.
```

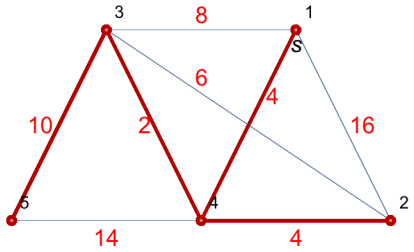
The highlighted graph above and the value in the green cell match the answer in the text.

7. Problem represented by a diagram.

```
g5 = Graph[{1 ↔ 2, 1 ↔ 3, 1 ↔ 4, 3 ↔ 5, 3 ↔ 4, 4 ↔ 5, 2 ↔ 4, 2 ↔ 3},
  VertexLabels → "Name",
  VertexCoordinates -> {{0, 0}, {1, -2}, {-2, 0}, {-1, -2}, {-3, -2}},
  EdgeWeight → {16, 8, 4, 10, 2, 14, 4, 6},
  Epilog → {{Text[Style["s", Medium], {0, -0.15}]},
    {Red, Text[Style["8", Medium], {-1, 0.15}]},
    {Red, Text[Style["10", Medium], {-2.7, -1}]},
    {Red, Text[Style["6", Medium], {-1, -0.5}]},
    {Red, Text[Style["2", Medium], {-1.6, -1}]},
    {Red, Text[Style["14", Medium], {-2, -2.18}]},
    {Red, Text[Style["4", Medium], {0, -2.18}]},
    {Red, Text[Style["16", Medium], {0.7, -1}]},
    {Red, Text[Style["4", Medium], {-0.25, -0.7}]}},
  ImageSize → 230, ImagePadding → 10]
```



```
FindSpanningTree[g5];
HighlightGraph[g5, %, GraphHighlightStyle → "Thick"]
```




```
size[g_] := With[{edges = EdgeList[FindSpanningTree[{g, 1}]]},
  Total[PropertyValue[{g, #}, EdgeWeight] & /@ edges]]
N[
 size[
  g5]]
```

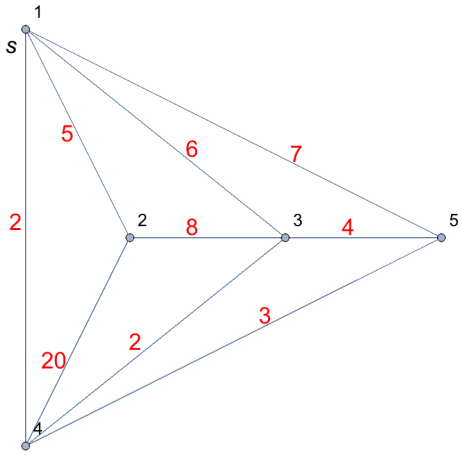
```
20.
```

The highlighted graph above and the value in green match the answer in the text.

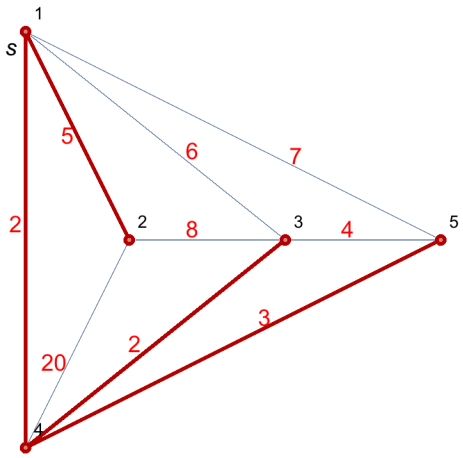
11. For the graph in problem 4, section 23.4.

The following graph is from problem 4 of the last section.

```
g4 = Graph[{1 ↔ 2, 2 ↔ 3, 3 ↔ 5, 1 ↔ 4, 4 ↔ 2, 1 ↔ 5, 3 ↔ 4, 4 ↔ 5, 1 ↔ 3},
  VertexLabels → "Name",
  VertexCoordinates -> {{0, 2}, {1, 0}, {2.5, 0}, {4, 0}, {0, -2}},
  EdgeWeight → {5, 8, 4, 2, 20, 7, 2, 3, 6},
  Epilog → {{Text[Style["s", Medium], {0 - .14, 1.85}]},
    {Red, Text[Style["2", Medium], {-0.1, 0.15}]},
    {Red, Text[Style["5", Medium], {0.4, 1}]},
    {Red, Text[Style["6", Medium], {1.6, 0.85}]},
    {Red, Text[Style["7", Medium], {2.6, 0.8}]},
    {Red, Text[Style["8", Medium], {1.6, 0.1}]},
    {Red, Text[Style["20", Medium], {0.27, -1.18}]},
    {Red, Text[Style["2", Medium], {1.05, -1}]},
    {Red, Text[Style["3", Medium], {2.3, -0.75}]},
    {Red, Text[Style["4", Medium], {3.1, 0.1}]}},
  ImageSize → 250, ImagePadding → 10]
```




```
FindSpanningTree[g4];
HighlightGraph[g4, %, GraphHighlightStyle → "Thick"]
```



```
size[g_] := With[{edges = EdgeList[FindSpanningTree[{g, 1}]]},
  Total[PropertyValue[{g, #}, EdgeWeight] & /@ edges]]
N[
 size[
  g4]]
```

```
12.
```

The highlighted graph above and the value in green match the answer in the text.